Bend-twist coupling in Bladed beam elements

Contents

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Updates in issue C

- Update regarding new options in Bladed 4.9

1 OVERVIEW

This document describes how bend-twist coupling effects can be account for in Bladed beam elements, in various Bladed 4.x versions.

Built-in Bladed model of bend-twist coupling due to shear centre offset from the neutral axis are described.

Additionally, the specification of user-defined bend-twist coupling terms is discussed.

2 BEND-TWIST COUPLING RELATIONSHIPS

2.1 Co-incident shear and neutral axes

This model is available in all Bladed 4.x versions

If the elastic centre and shear centre coincide, the constitutive relationship between strain and internal load for a beam element can be expressed as a diagonal matrix as shown below.

Note that this equation is formulated in the element local coordinate system (i.e. it is rotated according to blade structural twist, prebend and sweep).

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EA & & & | & & & \\ 0 & GA_y & & | & & Symm & \\ 0 & 0 & GA_z & | & & & \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & | & GI_x^* & & \\ 0 & 0 & 0 & | & 0 & EI_y & \\ 0 & 0 & 0 & | & 0 & 0 & EI_z \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} = \bar{\bar{C}} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

The 6x6 constitutive matrix is referred to in this document as $\bar{\bar{C}}$, where the double over-bar denotes the local element coordinate system.

2.2 Translational offset between neutral and shear axes

This model is used by default in Bladed 4.0 – 4.3, and is optional in 4.9+ by selecting "ignore blade shear centre axis orientation transformation" in Additional Items.

It is possible to define a translational offset between the neutral axis and the shear centre within the blade section, as illustrated in Figure 2-1.

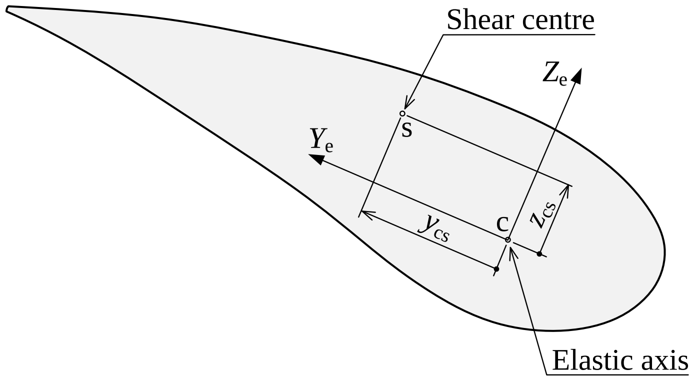

Figure 2-1: Shear centre offset from neutral axis

The translational offset between shear and neutral axes is taken into account using the following calculation, which transforms the shear properties onto the neutral axis position.

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \mathbf{1} & 0 \\ Y_S^T & \mathbf{1} \end{bmatrix} \left[\begin{array}{ccc|ccc} EA & & & & Symm & \\ 0 & GA_y & & & & \\ 0 & 0 & GA_z & & & \\ - & - & - & - & - & - \\ 0 & 0 & 0 & GI_x^* & & \\ 0 & 0 & 0 & 0 & EI_y & \\ 0 & 0 & 0 & 0 & 0 & EI_z \end{array}\right] \begin{bmatrix} \mathbf{1} & Y_S \\ 0 & \mathbf{1} \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Where

$$Y_S = \begin{bmatrix} 0 & 0 & 0 \\ -z_{cs} & 0 & 0 \\ y_{cs} & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

y_{cs} and z_{cs} define the position of the shear centre relative to the neutral axis.

Expanding the above expression gives the following constitutive relationship around the neutral axis. The effect of shear centre offset is to introduce additional coupling between shear strain and torsional moment, and between bending strain and shear force.

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \left[\begin{array}{ccc|ccc} EA & & & & Symm & \\ 0 & GA_y & & & & \\ 0 & 0 & GA_z & & & \\ - & - & - & - & - & - \\ 0 & -z_{cs}GA_y & y_{cs}GA_z & GI_x & & \\ 0 & 0 & 0 & 0 & EI_y & \\ 0 & 0 & 0 & 0 & 0 & EI_z \end{array}\right] \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Where $GI_x = GI_x{}^* + GA_y z_{cs}^2 + GA_z y_{cs}^2$

and $GI_x{}^*$ is the torsional stiffness defined around the shear (torsional) axis.

2.3 Translation and orientation offset between neutral and shear axes

This model is used as default from Bladed 4.4 onwards

From Bladed 4.4, the blade structural model was extended to account for the *orientation* difference between the elastic axis and the shear axis.

In general the elastic and shear axes are not parallel, so it can be important to take account of the orientation difference between them.

The orientation difference between the shear axis and the elastic axis is illustrated by the θ terms in Figure 2-2.

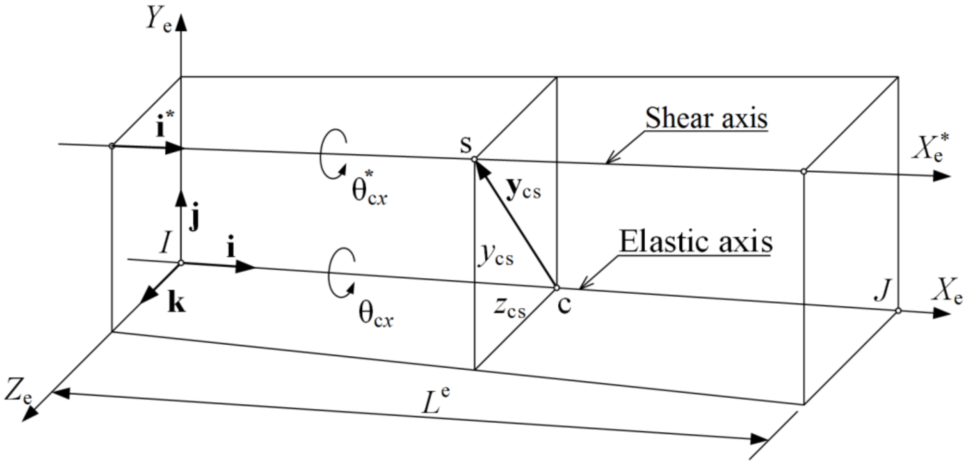

Figure 2-2: Orientation difference between shear and elastic axes.

The combined translational and orientation offset between shear and neutral axes is taken into account using the following calculation, which transforms the shear properties onto the neutral axis position.

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \mathbf{1} & 0 \\ Y_S^T & R_S^T \end{bmatrix} \left[\begin{array}{ccc|ccc} EA & & & & & \\ 0 & GA_y & & & Symm & \\ 0 & 0 & GA_z & & & \\ \hline 0 & 0 & 0 & GI_x^* & & \\ 0 & 0 & 0 & 0 & EI_y & \\ 0 & 0 & 0 & 0 & 0 & EI_z \end{array}\right] \begin{bmatrix} \mathbf{1} & Y_S \\ 0 & R_S \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

Where

$$R_s = \frac{1}{L^e} \begin{bmatrix} L_S^e & 0 & 0 \\ -\Delta y_{cs} & L^e & 0 \\ -\Delta \mathrm{z}_{cs} & 0 & L^e \end{bmatrix}$$

$$Y_S = \begin{bmatrix} 0 & 0 & 0 \\ -z_{cs} & 0 & 0 \\ y_{cs} & 0 & 0 \end{bmatrix}$$

Δy_{cs} and $\Delta \mathrm{z}_{cs}$ describe the change in position of the shear centre within the beam element, in order to describe the shear axis orientation.

The effect of shear centre translation and orientation offset is to introduce additional coupling between bending and torsional moments, resulting in the following constitutive relationship around the neutral axis.

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EA & & & | & & & \\ 0 & GA_y & & | & & Symm & \\ 0 & 0 & GA_z & | & & & \\ - & - & - & - & - & - & - \\ 0 & -z_{cs}GA_y & y_{cs}GA_z & | & GI_x & & \\ 0 & 0 & 0 & | & -\frac{\Delta y_{cs} EI_y}{L^e} & EI_y & \\ 0 & 0 & 0 & | & -\frac{\Delta z_{cs} EI_z}{L^e} & 0 & EI_z \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

where

$$GI_x = \left(\frac{L^e}{L_s^e}\right)^2 GI_x^* + GA_y z_{cs}^2 + GA_z y_{cs}^2 + \frac{\Delta y_{cs}^2 EI_y + \Delta z_{cs}^2 EI_z}{{L^e}^2}$$

And $GI_x{}^*$ is the torsional stiffness defined around the shear (torsional) axis.

This transformation results in extra bend-twist off–diagonal coupling terms, as well as a change to the torsional stiffness around the neutral axis.

2.4 User-defined bend-twist coupling

This model is available as an option from Bladed 4.3

The user can directly add extra off-diagonal terms to the constitutive matrix as shown

$$\bar{\bar{C}} = \begin{bmatrix} EA & & & | & & Symm & \\ 0 & GA_y & & | & & & \\ 0 & 0 & GA_z & | & & & \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & | & GI_x^* & & \\ 0 & 0 & 0 & | & C_{xy} & EI_y & \\ 0 & 0 & 0 & | & C_{xz} & C_{yz} & EI_z \end{bmatrix}$$

The transformations described in previous sections based on shear axis position relative to neutral axis are also applied, resulting in the following relationship in Bladed 4.3. This relationship can also be achieved in Bladed 4.9+ by selecting the option "ignore blade shear centre axis orientation transformation" in *Additional Items*.

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \mathbf{1} & 0 \\ Y_S^T & \mathbf{1} \end{bmatrix} \begin{bmatrix} EA & & & | & & Symm & \\ 0 & GA_y & & | & & & \\ 0 & 0 & GA_z & | & & & \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & | & GI_x^* & & \\ 0 & 0 & 0 & | & C_{xy} & EI_y & \\ 0 & 0 & 0 & | & C_{xz} & C_{yz} & EI_z \end{bmatrix} \begin{bmatrix} \mathbf{1} & Y_S \\ 0 & \mathbf{1} \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

The following transformation is used from Bladed 4.4+

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \mathbf{1} & 0 \\ Y_S^T & R_S^T \end{bmatrix} \begin{bmatrix} EA & & & | & & Symm & \\ 0 & GA_y & & | & & & \\ 0 & 0 & GA_z & | & & & \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & | & GI_x^* & & \\ 0 & 0 & 0 & | & C_{xy} & EI_y & \\ 0 & 0 & 0 & | & C_{xz} & C_{yz} & EI_z \end{bmatrix} \begin{bmatrix} \mathbf{1} & Y_S \\ 0 & R_S \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

2.4.1 Definition in Project Info

The explicit bend-twist coupling terms can be specified through Project Info.

You need enough entries for the number of elements in the blade – each node in the Bladed user interface defines the properties for both the end of one element and the beginning of the next, unless it is a split station so there must be 2N-2 entries.

```
MSTART EXTRA
TorsionEdgeCStiff * list of Cxz values
TorsionFlapCStiff * list of Cxy values
FlapEdgeCStiff * list of Cyz values
MEND
```

For example, for the demo_a turbine (which has 10 blade stations):

```
MSTART EXTRA
TorsionEdgeCStiff 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0
10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0
10000.0
TorsionFlapCStiff 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0
10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0
10000.0
FlapEdgeCStiff 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0
10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0 10000.0
MEND
```